Quantum corrections to the mass of self-dual vortices

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The mass shift induced by one-loop quantum fluctuations on self-dual ANO vortices is computed using heat kernel/generalized zeta function regularization methods.

PACS numbers: 03.70.+k,11.15.Kc,11.15.Ex

1. In this note we shall compute the one-loop mass shift for Abrikosov-Nielsen-Olesen self-dual vortices in the Abelian Higgs model. Non-vanishing quantum corrections to the mass of $N = 2$ supersymmetric vortices were reported during the last year in papers [1] and [2]. In the second paper, it was found that the central charge of the $N = 2$ SUSY algebra also receives a non-vanishing one-loop correction which is exactly equal to the one-loop mass shift; thus, one could talk about one-loop BPS saturation. This latter result fits in a pattern first conjectured in [3] and then proved in [4] for supersymmetric kinks. Recent work by the authors of the Stony Brook/Viena group, [5], unveils a similar kind of behaviour of supersymmetric BPS monopoles in $N = 2$ SUSY Yang-Mills theory. In this reference, however, it is pointed out that (2+1)-dimensional SUSY vortices behave not exactly in the same way as their (1+1)- and (3+1)-dimensional cousins. One-loop corrections in the vortex case are in no way related to an anomaly in the conformal central charge, contrarily to the quantum corrections for SUSY kinks and monopoles.

We shall focus, however, on the purely bosonic Abelian Higgs model and rely on the heat kernel/generalized zeta function regularization method that we developed in references [6], [7] and [8] to compute the one-loop shift to kink masses. Our approach profits from the high-temperature expansion of the heat function, which is compatible with Dirichlet boundary conditions in purely bosonic theories. In contrast, the application of a similar regularization method to the supersymmetric kink requires SUSY friendly boundary conditions, see [9]. We shall also encounter more difficulties than in the kink case due to the jump from one to two spatial dimensions.

Defining non-dimensional space-time variables, $x^\mu \to \frac{1}{ev}x^\mu$, and fields, $\phi \to v\phi = v(\phi_1 + i\phi_2)$, $A_\mu \to vA_\mu$, from the vacuum expectation value of the Higgs field v and the $U(1)$-gauge coupling constant e, the action for the Abelian Higgs model in (2+1)-dimensions reads:

$$S = \frac{v}{e}\int d^3x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi)^* D^\mu\phi - U(\phi,\phi^*)\right]$$

with $U(\phi,\phi^*) = \frac{\kappa}{8}(\phi^*\phi - 1)^2$. $\kappa = \frac{\lambda}{e^2}$ is the only classically relevant parameter and measures the ratio between the masses of the Higgs and vector particles; λ is the Higgs field self-coupling. For $\kappa = 1$ one finds self-dual vortices with quantized magnetic flux $g = \frac{2\pi l}{e}$, $l \in \mathbb{Z}$, and mass $M_V = \pi|l|v^2$ as the solutions of the first-order equations $D_1\phi \pm iD_2\phi = 0$, $F_{12} \pm \frac{1}{2}(\phi^*\phi - 1) = 0$, or,

$$(\partial_1\phi_1 + A_1\phi_2) \mp (\partial_2\phi_2 - A_2\phi_1) = 0 \qquad (1)$$

$$\pm(\partial_2\phi_1 + A_2\phi_2) + (\partial_1\phi_2 - A_1\phi_1) = 0 \qquad (2)$$

$$F_{12} \pm \frac{1}{2}(\phi_1^2 + \phi_2^2 - 1) = 0 \qquad (3)$$

with appropriate boundary conditions: $\phi^*\phi|_{S_\infty} = 1$, $D_i\phi|_{S_\infty} = (\partial_i\phi - iA_i\phi)|_{S_\infty} = 0$, that is, $\phi|_{S_\infty} = e^{il\theta}$ and $A_i|_{S_\infty} = -i\phi^*\partial_i\phi|_{S_\infty}$. In what follows, we shall focus on solutions with positive l: i.e., we shall choose the upper signs in the first-order equations.

2. L^2-integrable second-order fluctuations around a given vortex solution are still solutions of the first-order equations with the same magnetic flux if they belong to the kernel of the Dirac-like operator, $\mathcal{D}\xi(\vec{x}) = 0$, [10]

$$\mathcal{D} = \begin{pmatrix} -\partial_2 & \partial_1 & \psi_1 & \psi_2 \\ -\partial_1 & -\partial_2 & -\psi_2 & \psi_1 \\ \psi_1 & -\psi_2 & -\partial_2 + V_1 & -\partial_1 - V_2 \\ \psi_2 & \psi_1 & \partial_1 + V_2 & -\partial_2 + V_1 \end{pmatrix}$$

where $\xi^T(\vec{x}) = (a_1(\vec{x}), a_2(\vec{x}), \varphi_1(\vec{x}), \varphi_2(\vec{x}))$. We denote the vortex solution fields as $\psi = \psi_1 + i\psi_2$ and V_k, $k = 1, 2$. Assembling the small fluctuations around the solution $\phi(\vec{x}) = \psi(\vec{x}) + \varphi(\vec{x})$, $A_k(\vec{x}) = V_k(\vec{x}) + a_k(\vec{x})$ in a four column $\xi(\vec{x})$, the first component of $\mathcal{D}\xi$ gives the deformation of the vortex equation (3), whereas the third and fourth components are due to the respective deformation of the covariant holomorphy equations (2) and (1). The second component sets the background gauge $B(a_k, \varphi; \psi) = \partial_k a_k - (\psi_1\varphi_2 - \psi_2\varphi_1)$ on the fluctuations. The operators

$$\mathcal{H}^+=\begin{pmatrix} -\triangle+|\psi|^2 & 0 & -2\nabla_1\psi_2 & 2\nabla_1\psi_1 \\ 0 & -\triangle+|\psi|^2 & -2\nabla_2\psi_2 & 2\nabla_2\psi_1 \\ -2\nabla_1\psi_2 & -2\nabla_2\psi_2 & -\triangle+\frac{1}{2}(3|\psi|^2+2V_kV_k-1) & -2V_k\partial_k \\ 2\nabla_1\psi_1 & 2\nabla_2\psi_1 & 2V_k\partial_k & -\triangle+\frac{1}{2}(3|\psi|^2+2V_kV_k-1) \end{pmatrix}$$

$$\mathcal{H}^-=\begin{pmatrix} -\triangle+|\psi|^2 & 0 & 0 & 0 \\ 0 & -\triangle+|\psi|^2 & 0 & 0 \\ 0 & 0 & -\triangle+\frac{1}{2}(|\psi|^2+1)+V_kV_k & -2V_k\partial_k \\ 0 & 0 & 2V_k\partial_k & -\triangle+\frac{1}{2}(|\psi|^2+1)+V_kV_k \end{pmatrix} ,$$

are defined as $\mathcal{H}^+=\mathcal{D}^\dagger\mathcal{D}$ -the second order fluctuation operator around the vortex in the background gauge- and its partner $\mathcal{H}^-=\mathcal{D}\mathcal{D}^\dagger$.

One easily checks that $\dim\ker\mathcal{D}^\dagger=0$. Thus, the dimension of the moduli space of self-dual vortex solutions with magnetic charge l is the index of $\mathcal{D}$: $\mathrm{ind}\mathcal{D}=\mathrm{dimker}\mathcal{D}-\mathrm{dimker}\mathcal{D}^\dagger$. We follow Weinberg [10], using the background instead of the Coulomb gauge, to briefly determine $\mathrm{ind}\mathcal{D}$. The spectra of the operators $\mathcal{H}^+$ and $\mathcal{H}^-$ only differ in the number of eigen-functions belonging to their kernels. For topological vortices, we do not expect pathologies due to asymmetries between the spectral densities of $\mathcal{H}^+$ and $\mathcal{H}^-$ and thus $\mathrm{ind}\,\mathcal{D}=\mathrm{Tr}e^{-\beta\mathcal{H}^+}-\mathrm{Tr}e^{-\beta\mathcal{H}^-}$. See [11, 12] for the case of Chern-Simons-Higgs topological vortices.

The heat traces $\mathrm{Tr}e^{-\beta\mathcal{H}^\pm}=\mathrm{tr}\int_{\mathbb{R}^2}d^2\vec{x}\,K_{\mathcal{H}^\pm}(\vec{x},\vec{x};\beta)$ can be obtained from the kernels of the heat equations:

$$\left(\frac{\partial}{\partial\beta}\mathbb{I}+\mathcal{H}^\pm\right)K_{\mathcal{H}^\pm}(\vec{x},\vec{y};\beta)=0$$
$$K_{\mathcal{H}^\pm}(\vec{x},\vec{y};0)=\mathbb{I}\cdot\delta^{(2)}(\vec{x}-\vec{y})$$

Bearing in mind the structure $\mathcal{H}^\pm=-\triangle\mathbb{I}+\mathbb{I}+Q_k^\pm(\vec{x})\partial_k+V^\pm(\vec{x})$, one writes the heat kernels in the form:

$$K_{\mathcal{H}^\pm}(\vec{x},\vec{y};\beta)=C^\pm(\vec{x},\vec{y};\beta)K_{\mathcal{H}_0}(\vec{x},\vec{y};\beta)$$

with $C^\pm(\vec{x},\vec{x};0)=\mathbb{I}$. $K_{\mathcal{H}_0}(\vec{x},\vec{y};\beta)=\frac{e^{-\beta}}{4\pi\beta}\cdot\mathbb{I}\cdot e^{-\frac{|\vec{x}-\vec{y}|}{4\beta}}$ is the heat kernel for the Klein-Gordon operator $\mathcal{H}_0=(-\triangle+1)\mathbb{I}$, which is the second-order fluctuation operator around the vacuum in the Feynman-'t Hooft renormalizable gauge, the background gauge in the vacuum sector. $C^\pm(\vec{x},\vec{y};\beta)$ solve the transfer equations:

$$\Big\{\frac{\partial}{\partial\beta}\mathbb{I}+\frac{x_k-y_k}{\beta}(\partial_k\mathbb{I}-\frac{1}{2}Q_k^\pm)-\triangle\mathbb{I}+ +Q_k^\pm\partial_k+V^\pm\Big\}C^\pm(\vec{x},\vec{y};\beta)=0 \qquad (4)$$

The high-temperature expansions $C^\pm(\vec{x},\vec{y};\beta)=\sum_{n=0}^\infty c_n^\pm(\vec{x},\vec{y})\beta^n$, $c_0^\pm(\vec{x},\vec{x})=\mathbb{I}$, trade the PDE (4) by the recurrence relations

$$[n\mathbb{I}+(x_k-y_k)(\partial_k\mathbb{I}-\frac{1}{2}Q_k^\pm)]c_n^\pm(\vec{x},\vec{y})= =[\triangle\mathbb{I}-Q_k^\pm\partial_k-V^\pm]c_{n-1}^\pm(\vec{x},\vec{y}) \qquad (5)$$

among the coefficients with $n\geq 1$. Because

$$\mathrm{Tr}e^{-\beta\mathcal{H}^\pm}=\frac{e^{-\beta}}{4\pi\beta}\sum_{n=0}^\infty\sum_{a=1}^4\int d^2x\,[c_n]^\pm_{aa}(\vec{x},\vec{x})\beta^n= =\frac{e^{-\beta}}{4\pi\beta}\sum_{n=0}^\infty\beta^n\sum_{a=1}^4[c_n]^\pm_{aa}(\mathcal{H}^\pm) \qquad (6)$$

and $c_1^\pm(\vec{x},\vec{x})=-V^\pm(\vec{x})$, we obtain in the $\beta=0$ -infinite temperature- limit:

$$\mathrm{ind}\mathcal{D}=\frac{1}{4\pi}\mathrm{tr}\left\{c_1(\mathcal{H}^+)-c_1(\mathcal{H}^-)\right\}=\frac{1}{\pi}\int d^2xV_{12}(\vec{x})=2l$$

the dimension of the self-dual vortex moduli space is $2l$.

3. Standard lore in the semi-classical quantization of solitons tells us that the one-loop mass shift comes from the Casimir energy plus the contribution of the mass renormalization counter-terms: $\Delta M_V=\Delta M_V^C+\Delta M_V^R$. The vortex Casimir energy with respect to the vacuum Casimir energy is given formally by the formula:

$$\Delta M_V^C=\frac{\hbar m}{2}\left[\mathrm{STr}^*\left(\mathcal{H}^+\right)^{\frac{1}{2}}-\mathrm{STr}\left(\mathcal{H}_0\right)^{\frac{1}{2}}\right] ,$$

where $m=ev$ is the Higgs and vector boson mass at the critical point $\kappa=1$. We choose a system of units where $c=1$, but $\hbar$ has dimensions of length $\times$ mass. The "super traces" encode the ghost contribution to suppress the pure gauge oscillations: $\mathrm{STr}^*\left(\mathcal{H}^+\right)^{\frac{1}{2}}=\mathrm{Tr}^*\left(\mathcal{H}^+\right)^{\frac{1}{2}}-\mathrm{Tr}\left(\mathcal{H}^G\right)^{\frac{1}{2}}$ and $\mathrm{STr}\left(\mathcal{H}_0\right)^{\frac{1}{2}}=\mathrm{Tr}\left(\mathcal{H}_0\right)^{\frac{1}{2}}-\mathrm{Tr}\left(\mathcal{H}_0^G\right)$. The trace for the ghosts operators is purely functional: i.e., $\mathcal{H}^G=-\triangle+|\psi|^2$, $\mathcal{H}_0^G=-\triangle+1$ are ordinary -non-matricial- Schrodinger operators. The star means that the $2n$ zero eigenvalues of $\mathcal{H}^+$ must be subtracted because zero modes only enter at two-loop order.

In a minimal subtraction renormalization scheme, one adds the counter-terms $\mathcal{L}^S_{c.t.}=\hbar mI\left[|\phi|^2-1\right]$, $\mathcal{L}^A_{c.t.}=-\frac{1}{2}\hbar mIA_\mu A^\mu$ with $I=\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{1}{\sqrt{\vec{k}\cdot\vec{k}+1}}$ to cancel the divergences up to the one-loop-order that arises in the Higgs tadpole and two-point function, and in the two-point functions of the Goldstone and vector bosons. Finite renormalizations are adjusted in such a way that the critical point $\kappa=1$ is reached at first-order in the

loop expansion. Therefore, the contribution of the mass renormalization counter-terms to the vortex mass is:

$$\Delta M_V^R = \Delta M_{c.t.}^S + \Delta M_{c.t.}^A = \hbar\, m\, I\, \Sigma(\psi, V_k)$$

where $\Sigma(\psi, V_k) = \int dx^2\, [(1 - |\psi|^2) - \frac{1}{2} V_k V_k]$.

We regularize both ΔM_V^C and ΔM_V^R by means of generalized zeta functions. From the spectral resolution of a Fredholm operator $\mathcal{H}\xi_n = \lambda_n \xi_n$, one defines the generalized zeta function as the series $\zeta_{\mathcal{H}}(s) = \sum_n \frac{1}{\lambda_n^s}$, which is a meromorphic function of the complex variable s. We can then hope that, despite their continuous spectra, our operators fits in this scheme, and write:

$$\begin{aligned}
\Delta M_V^C(s) &= \frac{\hbar\mu}{2}\left(\frac{\mu^2}{m^2}\right)^s \left\{ \left(\zeta_{\mathcal{H}^+}(s) - \zeta_{\mathcal{H}_G^+}(s)\right) + \right. \\
&\quad \left. + \left(\zeta_{\mathcal{H}_0^G}(s) - \zeta_{\mathcal{H}_0}(s)\right) \right\} \\
\Delta M_V^R(s) &= \frac{\hbar}{mL^2} \zeta_{\mathcal{H}_0}(s) \Sigma(\psi, V_k)
\end{aligned}$$

where $\zeta_{\mathcal{H}_0}(s) = \frac{m^2 L^2}{4\pi} \frac{\Gamma(s-1)}{\Gamma(s)}$ and μ is a parameter of inverse length dimensions. Note that $\Delta M_V^C = \lim_{s \to -\frac{1}{2}} \Delta M_V^C(s)$, $\Delta M_V^R = \lim_{s \to \frac{1}{2}} \Delta M_V^R(s)$ and $I = \lim_{s \to \frac{1}{2}} \frac{1}{2m^2 L^2} \zeta_{\mathcal{H}_0}(s)$.

4. Together with the high-temperature expansion the Mellin transform of the heat trace shows that

$$\zeta_{\mathcal{H}}(s) = \frac{1}{\Gamma(s)} \sum_{n=0}^{\infty} \int_0^1 d\beta\, \beta^{s+n-2} c_n(\mathcal{H}) e^{-\beta} + \frac{1}{\Gamma(s)} B_{\mathcal{H}}(s)$$

is the sum of meromorphic and entire -$B_{\mathcal{H}}(s)$- functions of s. Neglecting the entire parts and keeping a finite number of terms N_0 in the asymptotic series for $\zeta_{\mathcal{H}}(s)$, we find the following approximations for the generalized zeta functions concerning our problem:

$$\begin{aligned}
\zeta_{\mathcal{H}^+}(s) - \zeta_{\mathcal{H}_0}(s) &\simeq \sum_{n=1}^{N_0} \sum_{a=1}^{4} [c_n]_{aa}(\mathcal{H}^+) \cdot \frac{\gamma[s+n-1, 1]}{4\pi\Gamma(s)} \\
\zeta_{\mathcal{H}_0^G}(s) - \zeta_{\mathcal{H}^G}(s) &\simeq -\sum_{n=1}^{N_0} c_n(\mathcal{H}^G) \cdot \frac{\gamma[s+n-1, 1]}{4\pi\Gamma(s)} \quad ;
\end{aligned}$$

$\gamma[s+n-1, 1] = \int_0^1 d\beta\, \beta^{s+n-2} e^{-\beta}$ is the incomplete Gamma function, with a very well known meromorphic structure. Contrarily to the (1+1)-dimensional case, the value $s = -\frac{1}{2}$ for which we shall obtain the Casimir energy is not a pole.

Writing $\bar{c}_n = \sum_{a=1}^{4} [c_n]_{aa}(\mathcal{H}^+) - c_n(\mathcal{H}^G)$, the contribution of the first coefficient to the Casimir energy

$$\Delta M_V^{(1)C}(s) \simeq \frac{\hbar}{2}\mu \left(\frac{\mu^2}{m^2}\right)^s \bar{c}_1 \cdot \frac{\gamma[s, 1/2]}{4\pi\Gamma(s)}$$

is finite at the $s \to -\frac{1}{2}$ limit

$$\Delta M_V^{(1)C}(-1/2) \simeq -\frac{\hbar m}{4\pi} \Sigma(\psi, V_k) \cdot \frac{\gamma[-1/2, 1]}{\Gamma(1/2)}$$

and exactly cancels the contribution of the mass renormalization counter-terms -also finite for $s = \frac{1}{2}$-:

$$\begin{aligned}
\Delta M_V^R(s) &\simeq \frac{\hbar m}{4\pi} \cdot \Sigma(\psi, V_k) \cdot \frac{\gamma[s-1, 1]}{\Gamma(s)} \\
\Delta M_V^R(1/2) &\simeq \frac{\hbar m}{4\pi} \cdot \Sigma(\psi, V_k) \cdot \frac{\gamma[-1/2, 1]}{\Gamma(1/2)} \, .
\end{aligned}$$

Subtracting the contribution of the $2l$ zero modes we finally obtain the following formula for the vortex mass shift:

$$\begin{aligned}
\Delta M_V &= \frac{\hbar m}{2} \lim_{s \to -\frac{1}{2}} \left[-2l \frac{\gamma[s, 1]}{\Gamma(s)} + \sum_{n=2}^{N_0} \bar{c}_n \frac{\gamma[s+n-1, 1]}{4\pi\Gamma(s)} \right] \\
&= -\frac{\hbar m}{16\pi^{\frac{3}{2}}} \left[-2l\gamma[-\frac{1}{2}, 1] + \sum_{n=2}^{N_0} \bar{c}_n \gamma[n - 3/2, 1] \right] \qquad (7)
\end{aligned}$$

5. Computation of the coefficients of the asymptotic expansion is a difficult task; e.g. the second coefficient

$$\begin{aligned}
c_2^+(\vec{x}, \vec{x}) &= -\frac{1}{6} \triangle V^+(\vec{x}) + \frac{1}{12} Q_k^+(\vec{x}) Q_k^+(\vec{x}) V^+(\vec{x}) - \\
&- \frac{1}{6} \partial_k Q_k^+(\vec{x}) V^+(\vec{x}) + \frac{1}{6} Q_k^+(\vec{x}) \partial_k V^+(\vec{x}) + \frac{1}{2} [V^+]^2(\vec{x})
\end{aligned}$$

Defining the partial derivatives of the coefficients at $\vec{y} = \vec{x}$ as

$${}^{(\alpha_1,\alpha_2)}C_n^{ij}(\vec{x}) = \lim_{\vec{y} \to \vec{x}} \frac{\partial^{\alpha_1+\alpha_2} [c_n]_{ij}(\vec{x}, \vec{y})}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2}}$$

we write their recurrence relations

$$\begin{aligned}
(k + \alpha_1 + \alpha_2 + 1)\, {}^{(\alpha_1,\alpha_2)}C_{k+1}^{ip}(\vec{x}) &= {}^{(\alpha_1+2,\alpha_2)}C_k^{ip}(\vec{x}) + {}^{(\alpha_1,\alpha_2+2)}C_k^{ip}(\vec{x}) - \\
&- \sum_{j=1}^{n} \sum_{r=0}^{\alpha_1} \sum_{t=0}^{\alpha_2} \binom{\alpha_1}{r} \binom{\alpha_2}{t} \left[\frac{\partial^{r+t} Q_1^{ij}}{\partial x_1^r \partial x_2^t} {}^{(\alpha_1-r+1,\alpha_2-t)}C_k^{jp}(\vec{x}) + \frac{\partial^{r+t} Q_2^{ij}}{\partial x_1^r \partial x_2^t} {}^{(\alpha_1-r,\alpha_2-t+1)}C_k^{jp}(\vec{x}) \right] + \\
&+ \frac{1}{2} \sum_{j=1}^{n} \sum_{r=0}^{\alpha_1-1} \sum_{t=0}^{\alpha_2} \alpha_1 \binom{\alpha_1-1}{r} \binom{\alpha_2}{t} \frac{\partial^{r+t} Q_1^{ij}}{\partial x_1^r \partial x_2^t} {}^{(\alpha_1-1-r,\alpha_2-t)}C_{k+1}^{jp}(\vec{x}) +
\end{aligned}$$

$$+\frac{1}{2}\sum_{j=1}^{n}\sum_{r=0}^{\alpha_2-1}\sum_{t=0}^{\alpha_1}\alpha_2\binom{\alpha_2-1}{r}\binom{\alpha_1}{t}\frac{\partial^{r+t}Q_2^{ij}}{\partial x_1^t\partial x_2^r}{}^{(\alpha_1-t,\alpha_2-1-r)}C_{k+1}^{jp}(\vec{x})-$$
$$-\sum_{j=1}^{n}\sum_{r=0}^{\alpha_2}\sum_{t=0}^{\alpha_1}\binom{\alpha_1}{t}\binom{\alpha_2}{r}\frac{\partial^{r+t}V^{ij}}{\partial x_1^t\partial x_2^r}{}^{(\alpha_1-t,\alpha_2-r)}C_k^{jp}(\vec{x})$$

starting from ${}^{(\beta,\gamma)}C_0^{jp}(\vec{x})$.

We notice that $[c_n]_{jp}(\vec{x}) = {}^{(0,0)}C_n^{jp}(\vec{x})$ and thus $[c_n]_{ii}(\mathcal{H}) = \int_{-\infty}^{\infty} d^2x[c_n]_{ii}(\vec{x})$.

Things are easier if we apply these formulae to cylindrically symmetric vortices. The ansatz $\phi(r,\theta) = f(r)e^{il\theta}$ and $rA_\theta(r,\theta) = l\alpha(r)$ plugged into the first-order equations leads to:

$$\frac{1}{r}\frac{d\alpha}{dr} = \mp\frac{1}{2l}(f^2-1) \quad , \quad \frac{df}{dr} = \pm\frac{l}{r}f(r)[1-\alpha(r)] \quad . \quad (8)$$

Solutions of (8) with the boundary conditions $\lim_{r\to\infty} f(r) = 1$, $\lim_{r\to\infty}\alpha(r) = 1$, zeroes of the Higgs and vector fields at the origin, $f(0) = 0$, $\alpha(0) = 0$, and integer magnetic flux, $eg = -\int_{r=\infty} d\theta A_\theta = 2\pi l$, can be found by a mixture of analytical and numerical methods [13]. Henceforth, we shall focus on the case $l = 1$.

The heat kernel coefficients depend on successive derivatives of the solution. This dependence can increase the error in the estimation of these coefficients because we handle an interpolating polynomial as the numerically generated solution, and the derivation of such a polynomial introduces inaccuracies. It is thus of crucial importance to use the first-order differential equations (8) in order to eliminate the derivatives of the solution and write the coefficients as expressions depending only on the fields. The recurrence formula now gives the coefficients of the asymptotic expansion in terms of $f(r)$ and $\alpha(r)$, e.g.:

$$\sum_{i=1}^{4}[c_1]_{ii}(r,\theta) = 5 - \frac{2\,\alpha(r)^2}{r^2} - 5\,f(r)^2$$
$$\sum_{i=1}^{4}[c_2]_{ii}(r,\theta) = \frac{1}{12\,r^4}[37\,r^4 + 4\,\alpha(r)^4 - 8\,r^2\,\left(-7+8\,r^2\right)\,f(r)^2 +$$
$$+27\,r^4\,f(r)^4 + 8\,r^2\,\alpha(r)\,\left(1-14\,f(r)^2\right) +$$
$$+8\,\alpha(r)^2\,\left(-2-3\,r^2+9\,r^2\,f(r)^2\right)]$$
$$\sum_{i=1}^{4}[c_3]_{ii}(r,\theta) = \frac{1}{120\,r^6}[-4\,\alpha(r)^6 - 28\,r^2\,\alpha(r)^3\,\left(2+5\,f(r)^2\right) +$$
$$+4\,\alpha(r)^4\,\left(20+9\,r^2+32\,r^2\,f(r)^2\right) - 2\,r^2\,\alpha(r)\,\left(-4\,\left(16+9\,r^2\right) +\right.$$
$$\left. +\left(32+331\,r^2\right)\,f(r)^2 + 57\,r^2\,f(r)^4\right) + \alpha(r)^2\,\left(-256-144\,r^2\right.$$
$$\left.-117\,r^4 + 2\,r^2\,\left(56+183\,r^2\right)\,f(r)^2 + 99\,r^4\,f(r)^4\right) + r^4\,\left(-16+\right.$$
$$+151\,r^2 + \left(392-321\,r^2\right)\,f(r)^2 + \left(-20+199\,r^2\right)\,f(r)^4$$
$$\left.-29\,r^2\,f(r)^6\right)] \quad .$$

Plugging in these expressions the partially analytical partially numerical solution for $f(r)$ and $\alpha(r)$, it is possible to compute the coefficients -also for the ghost operator via similar but simpler formulae- and integrate numerically them in the whole plane. Thus, formula (7)

$$\frac{\Delta M_V}{\hbar m} = \frac{-1}{16\pi^{\frac{3}{2}}}\sum_{n=2}^{N_0}\bar{c}_n\gamma[-\frac{3}{2}+n,1] - \frac{1}{\sqrt{\pi}}$$

provides us with the one-loop vortex mass shift, where we recall that

$$\bar{c}_n = \sum_{a=1}^{4}[c_n]_{aa}(\mathcal{H}^+) - c_n(\mathcal{H}^G) \quad .$$

The results are shown in the Table I:

TABLE I: Seeley Coefficients and Mass Shift

n	$\sum_{i=1}^4 c_n^{ii}(\mathcal{H}^+)$	$c_n(\mathcal{H}^G)$	N_0	$\Delta M_V(N_0)/\hbar m$
2	30.3513	2.677510	2	-1.02814
3	13.0289	0.270246	3	-1.08241
4	4.24732	0.024586	4	-1.09191
5	1.05946	0.001244	5	-1.09350
6	0.207369	0.000013	6	-1.09373

The final value for the vortex mass at one-loop order is:

$$M_V = m\left(\frac{\pi v}{e} - 1.09373\hbar\right) + o(\hbar^2).$$

The convergence up to the sixth order in the asymptotic expansion is very good. We have no means, however, of estimating the error. In the case of $\lambda(\phi)_2^4$ kinks we found agreement between the result obtained by this method and the exact result up to the fourth decimal figure, see [6] .

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